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# The Dynamics of Incentive Contracts

by
Jean-Jacques Laffont*
and
Jean Tirole**

W.P. #397

July 1985
Revised March 1986

massachusetts
institute of
technology

50 memorial drive
cambridge, mass. 02139

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*Universite de Toulouse, GREMAQ and CERAS,
visiting the University of Southern California.

**Massachusetts Institute of Technology.

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## Abstract

The paper studies a simple two-period principal/agent model in which the principal updates the incentive scheme after observing the agent's first-period performance. The agent has superior information about his ability. The principal offers a first period incentive scheme and observes some measure of the agent's first-period performance (cost or profit), which depends on the agent's ability and (unobservable) first-period effort. The relationship is entirely run by short-term contracts. In the second-period the principal updates the incentive scheme and the agent is free to accept the new incentive scheme or to quit.

The strategies are required to be perfect and updating of the principal's beliefs about the agent's ability follows Bayes' rule.

The central theme of the paper is that the ratchet effect leads to much pooling in the first period. First, for any first-period incentive scheme, there exists no separating equilibirum. Second, when the uncertainty about the agent's ability is small, the optimal scheme must involve a large amount of pooling. The paper also gives necessary and sufficient conditions for the existence of partition equilibria.

## 1. Introduction.

In Laffont and Tirole (1984) we showed how cost observability helps a regulator controlling a private or public firm when both adverse selection and moral hazard affect their relationship. In particular, we showed that under cost observability, there is a trade-off between revelation of information through the selection of an incentive contract, and efficient ex-post production. The optimal sharing of overruns was obtained. The analysis was carried out in a one period model. Of course, relationships between firms and regulators are often repeated. The purpose of this paper is to extend our analysis of incentive contracts to a dynamic framework.[1]

Section 2 presents the model, which is a simplified version of our one-

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[1]Baron and Besanko (1984) and Roberts (1983) have studied long term relationships with full commitment under various assumptions concerning the type of adverse selection. One main result is that, in the independent case (each period a new independent variable of adverse selection is drawn), the optimal mechanism restores the first best from the second period on and all the inefficiency occurs in period 1. Intuitively this is because at the time the contract is signed, the agent does not really have an informational advantage concerning period 2 on. In the perfect correlation case (a variable of adverse selection is drawn at the beginning for the whole relationship), the optimal mechanism is the optimal static mechanism.

Baron and Besanko (1985) study multiperiod incentive schemes under adverse selection for alternative assumptions about commitment. The main focus of their paper is the "fairness case", in which the regulator has a limited commitment in the second period: he must be "fair" to the which is committed to stay. An interesting result of their analysis is that separation is feasible in the first period, contrary to the case of no commitment at all (see Proposition 1).

Freixas, Guesnerie and Tirole (1985) have studied repeated adverse selection in the perfect correlation case with no commitment but no moral hazard. Their analysis focuses on optimal linear incentive schemes. (see also the literature on the ratchet effect using nonoptimal mechanisms Weitzman (1980), Holmstrom (1982a)). Furthermore, it considers only the two-type case, which leads to a much weaker emphasis on pooling than the continuum of types framework envisioned in this paper.

firm, earlier model. Each period a very valuable indivisible project must be carried out. The ex-post cost of the project depends on two variables: the firm's intrinsic efficiency, and its level of "effort" during the period. The firm's efficiency is known to the firm, but not to the regulator. The firm's effort is not observed by the regulator either. The incentive scheme (transfer to the firm) thus depends on observed cost only. For reference, the optimal regulatory scheme under full information about the firm is derived.

Section 3 solves for the optimal incentive scheme in the static (one period) incomplete information case, and sets up the dynamic framework. For the rest of the paper, we consider the two-period version of the basic model. The regulator chooses a first-period incentive scheme, which specifies a transfer to the firm as a function of the first-period observed cost. The firm reacts to this incentive scheme by choosing a first-period level of effort. The regulator then rewards the firm and updates his beliefs about the latter's efficiency using the cost observation. In the second-period the regulator chooses the optimal regulatory scheme given the posterior beliefs, and the firm chooses a second-period effort and is rewarded.

We are thus assuming that the regulator cannot commit himself not to use in the second period the information conveyed by the firm's first-period performance. This assumption, which, we believe, is reasonable in a wide range of applications, certainly deserves comment. The simplest way to motivate it is the changing principal framework. For instance, the current administration cannot bind future ones[2] (the changing principal framework also

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[2]In this interpretation the discount factor of the principal is likely to be lower than the firm's. This feature can be embodied at no cost to our model, and reinforces our central theme that pooling is very likely in ratcheting situations (the principal then finds it even more costly to induce some separation).

applies to planning models). Non commitment situations are also very common in relationships between private parties, and in particular in organizations. Contracts are costly to write and contingencies are often hard to foresee, which gives rise to the allocation of discretion to some members of the organization (as emphasized by Coase [1937], Simon [1951], Williamson [1975, 1985], and Grossman-Hart [1985]). The existence of discretion creates scope for the parties who exercise this discretion to use the information revealed by the other parties.[3] (We have in mind, for example, the case of new tasks or new technologies which are costly or impossible to describe in advance, and such that the agent's ability to handle them is positively correlated with this ability to handle current tasks or technologies). As the theory of incomplete contracts is still in its infancy, we prefer to couch the model in a regulatory framework to motivate non-commitment.

The focus of this paper is the ratchet effect: an agent with a high performance today will tomorrow face a demanding incentive scheme. He should thus be reluctant to convey favorable information early in the

---

[3]As Katz-Kahn [1978] observe;

> ... there is the tradition among workers, and it is not without some factual basis, that management cannot be relied upon to maintain a high rate of pay for those making considerably more than the standard and that their increased efforts will only result in their "being sweated." There is, then, the temporal dimension of whether the piece rates that seem attractive today will be maintained tomorrow if individual productivity increases substantially. [p. 411]

For an interesting description of problems created by piece rates, see Gibbons [1985].

relationship.[4] The purpose of the paper is to formalize and confirm this intuition. Indeed, we show that, with a continuum of potential abilities for the agent, and for any first-period incentive scheme, no separating equilibrium exists. Furthermore, when the range of potential abilities is small, the principal cannot do better in the first period than imposing "much pooling" (in a sense to be defined later). This holds even when, in the static framework, full separation is optimal. These two results are derived in section 4. Section 5 exhibits some further results when uncertainty is not small. In particular, it gives necessary and sufficient conditions for the existence of partition equilibria.

## 2. The Model.

We consider a two-period model in which a firm (the agent) must, each period, realize a project with a cost structure:

$$c_t = \beta - e_t \qquad t = 1, 2,$$

where $e_t$ is the level of effort performed by the firm's manager in period t, and $\beta$ is a parameter known only by the manager.

Each period the manager's utility level is $s-\phi(e)$, where s is the net (i.e., in addition to cost) monetary transfer he receives from the regulator

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[4] The behavior is at variance with that described in Holmström [1982]. This is not surprising, since our assumptions differ from his in at least three respects. First, Holmström assumes that the efficiency parameter is unknown to both the principal and the agent. Second, the agent's cost (or profit) is observable, but not verifiable (so $s_t$ cannot be contingent on $c_t$). Third, Holmstrom's agent faces a competitive market, in which the transfer depends on past performance, while our agent's second-period outside opportunities are independent of the first-period outcome. A systematic study of the effect of these points of departure between the two models would be worthwhile, but is out of the scope of this paper. Let us simply notice that as an outcome, the agent has fairly different behaviors in the two set-ups. Holmstrom's agent tries to prove he is efficient while ours may try to prove the contrary.

and $\psi(e)$ is his disutility of effort ($\psi' > 0$, $\psi'' > 0$). For simplicity, we will allow any level of effort in R, and just require that $\psi'$ is bounded below by $k \in (0,1)$. Also, we assume that $\psi'$ goes to infinity when e goes to infinity and we define $e^*$ by $\psi'(e^*) \equiv 1$. Finally, let $\delta$ be the discount factor common to all parties (differences in discount factors could be costlessly introduced).

The regulator (principal) observes cost but not the effort level or the value of the parameter $\beta$. He has some prior cumulative distribution function $F_1(\beta)$ on $[\underline{\beta}, \bar{\beta}]$. We will assume that $F_1$ has a density $f_1$ and that $f_1$ is differentiable and bounded away from zero on $[\underline{\beta}, \bar{\beta}]$. The probability distribution should be interpreted as an objective probability that both the principal (the regulator) and the agent (the firm's manager) share before date 1.

Let u be, each period, the social utility of the project, which can be viewed for simplicity as a public good, i.e., as not sold on the market.[5] The gross payment made by the planner to the firm is s+c. We assume that there is a distortionary cost $\lambda > 0$ incurred to raise each unit of money (through excise taxes for example).

Consumers' welfare in period t is:

$$u - (1+\lambda)(s_t+c_t) \ .$$

Under perfect information ($\beta = \underline{\beta} = \bar{\beta}$), a utilitarian regulator would solve, each period t:

$$(2.1) \qquad \max_{s_t, e_t} \{u - (1+\lambda)(s_t+\beta-e_t) + s_t - \psi(e_t)\}$$

---

[5]We will assume that in the static framework it is always worth carrying out the project. It is sufficient to take u large enough. This assumption enables us to avoid the issue of determining the cut-off level above which firms do not operate in the one-period framework. In Fact 2 below, we will however consider the more general case in which the cut-off point is not necessarily $\bar{\beta}$.

(2.2) s.t. $s_t - \psi(e_t) \geq 0$ .

The individual rationality constraint (2.2) says that the utility level of the firm's manager must be positive to obtain his participation (the perfect information problem being stationary, the allocation, and therefore, the constraint will be the same at each period).

The optimal solution is characterized by

(2.3) $s_t = \psi(e_t)$

(2.4) $e_t = e^*$, where $\psi'(e^*) \equiv 1$ .

The individual rationality constraint is binding each period because there is a social loss due to transfers and the marginal disutility of effort is equated to marginal cost savings.

As a preliminary, we solve the static regulation problem under imperfect information, the dynamic problem with commitment and we set up the dynamic problem under no-commitment.

## 3. Preliminary analysis.

### a) Static regulation.

The regulator has two observables, namely the level of net transfer, s, and the level of cost, c. In general, the contract will be a function s(c) specifying the transfer received by the manager for each value of observed cost. Let us make the following familiar assumption on the prior distribution:

Assumption M: $\frac{F_1}{f_1}$ is non-decreasing.

Assumption M, which says that the hazard rate is monotone, is commonly made

in the incentives literature and is satisfied by many distributions. Assumption M is not required for the main propositions of the paper (it is used only in Proposition 4). We make this assumption to give separation its best chance: as is well-known, M ensures that the principal does not want to induce any bunching in the static case.

Appendix A rigorously characterizes the solution of the static incentive problem under asymmetric information. We here give an informal treatment of the main aspects.

First, the firm's profit $\Pi(\beta) = \max_{c} \{s(c) - \psi(\beta-c)\}$ is a non-increasing function of $\beta$, since a more efficient firm can always produce at the same cost as a less efficient firm by working less. From the "envelope theorem", its derivative is equal to $\left(-\psi'(e(\beta))\right)$ where $e(\beta)$ is the optimal effort of a firm with type $\beta$ when it faces the incentive scheme $s(\cdot)$. This tells us exactly how fast the firm's rent $\Pi$ must grow with productivity for a given effort schedule (for the effort schedule to be implementable, a second-order condition is required as well; but we ignore this condition, which is checked in the Appendix). To maximize the social welfare function

$$\int_{\underline{\beta}}^{\bar{\beta}} \{u - (1+\lambda)(s+\beta-e) + s-\psi(e)\} f_1(\beta) d\beta,$$

the regulator must trade off two conflicting objectives: 1) production efficiency, which requires that the marginal disutility of income $\psi'\left(e(\beta)\right)$ be equal to one for all $\beta$ and 2) the elimination of costly rents $\Pi(\beta)$. To reduce rents for the more efficient firms, the regulator can -- and will -- encourage less effort than in the first best.

The following simple results summarize the static analysis that is relevant to our model; they are special cases of results in Laffont-Tirole

(1984) (which allows a choice of scale and cost uncertainty) ($e^*(\beta)$ denote firm $\beta$'s effort for the optimal incentive scheme):

Fact 1: Under full information about the firm's type ($\beta = \underline{\beta} = \bar{\beta}$), the firm obtains no rent, and its effort is optimal: $\psi'(e^*(\beta)) = 1$.

Fact 2: If $\tilde{\beta}$ denotes the supremum in $[\underline{\beta}, \bar{\beta}]$ of types that are willing to participate (i.e., not to exercise their exit option) for the optimal incentive scheme, then $\Pi(\tilde{\beta}) = 0$.

Fact 3: There is no distortion for the most efficient firm: $\psi'(e^*(\underline{\beta})) = 1$.

Fact 4: Under assumption M and if $\psi''' > 0$, $e^*(\beta)$ is non-increasing. This implies that the optimal incentive scheme fully separates the various types. Together with fact 3, this also implies that for all $\beta$, $\psi'(e^*(\beta)) < 1$. Furthermore, for an "unexpected type" $\beta < \underline{\beta}$, firm $\beta$'s rent for one optimal scheme (which allows only costs in the expected range) is

$$\Pi(\beta) = \Pi(\underline{\beta}) + \psi(e^*) - \psi(e^*-(\underline{\beta}-\beta)).$$

Fact 1 was proven earlier. Facts 2 and 3 are familiar from the incentive literature. Fact 4 is used only to prove Proposition 4. Fact 1 contains the earlier observation that the agent does not enjoy any rent when the principal has full information. Fact 2 says that under asymmetric information the least productive of the active types does not obtain any rent. Fact 3 is the classic "no distortion at the top result." Fact 4 simply says that under assumption M, the optimal effort is suboptimal. It also gives the rent for an unexpected efficient type, who is forced to mimic the behavior of the

$\beta$ type.

b) Dynamic Optimization with Commitment.

In this subsection only, we consider the case in which the regulator is able to commit himself to an intertemporal incentive scheme. In particular, he can commit himself to a second period scheme which is not optimal given his information. Let us notice that the regulator offers twice the same scheme (similar reasonings can be found in Baron-Besanko (1984) and Roberts (1983)). Thus commitment in this model eliminates dynamics and ratcheting.

Let $\{t_1(\beta), c_1(\beta)\}$ and $\{t_2(\beta), c_2(\beta)\}$ denote the first- and second-period transfers and costs for firm $\beta$ under commitment. For both the firm and the regulator, this dynamic outcome is equivalent to the stationary (or static) random mechanism which gives

$$\{t_1(\beta),\ c_1(\beta)\} \quad \text{with probability} \quad \frac{1}{1+\delta}$$

and

$$\{t_2(\beta),\ c_2(\beta)\} \quad \text{with probability} \quad \frac{\delta}{1+\delta}$$

to a firm of type $\beta$. By construction, this static mechanism is incentive compatible. Since the objective functions are concave in t and c in turn, this random static mechanism is dominated by the following deterministic incentive mechanism:

$$\left\{t(\beta) = \frac{1}{1+\delta}\, t_1(\beta) + \frac{\delta}{1+\delta}\, t_2(\beta)\ ,\ c(\beta) = \frac{1}{1+\delta}\, c_1(\beta) + \frac{\delta}{1+\delta}\, c_2(\beta)\right\}$$

in both periods. Thus, the previous dynamic incentive scheme was not optimal. So it is optimal for the regulator to commit himself not to use the information revealed by the firm in the first period.

c) No commitment: presentation of the dynamic game.

We will assume in the rest of the paper that the regulator cannot commit himself to a second-period incentive scheme. He chooses the second-period incentive scheme optimally given his beliefs about the firm's type at that date. As explained above, these beliefs depend on the first-period cost, and on the firm's equilibrium first-period strategy (the firm's actual choice of effort cannot be observed by the regulator). For any first-period incentive scheme $s_1(c_1)$, each firm $\beta$ chooses a level of effort taking into account both the effect on the first-period reward and on the regulator's inference about its efficiency. Lastly, the regulator chooses the first-period incentive scheme knowing that the firm will take a dynamic perspective.

We allow the firm to quit the relationship (and to obtain its reservation utility 0) at any moment. Let $\chi_t = 1$ if the firm accepts the incentive scheme at t, and $\chi_t = 0$ if it quits.

To summarize, the regulator's strategies are incentive schemes $\{s_1(c_1), s_2(c_2)\}$ and the firm's strategies are effort levels and acceptance choices $\{(e_1, \chi_1), (e_2, \chi_2)\}$ conditional on the firm's type. These strategies must form a perfect Bayesian equilibrium: P1) $(e_2, \chi_2)$ is optimal for the firm given $s_2(\cdot)$, P2) $s_2$ is optimal for the regulator given his beliefs $F_2(\cdot | c_1)$, P3) $(e_1, \chi_1)$ is optimal for the firm given $s_1(\cdot)$ and the fact that the regulator's second-period scheme depends on $c_1$, P4) $s_1$ is optimal for the regulator given subsequent strategies, and B) $F_2(\cdot | c_1)$ is derived from the prior $F_1$, the firm's strategy given by P3) and the observed cost c, using Bayes' rule[6].

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[6]See Kreps-Wilson (1982).

A continuation equilibrium is a set of strategies and updating rule that satisfies P1), P2), P3) and B). In other words, it is an equilibrium for an exogenously given first-period incentive scheme.

## 4. Ratcheting and pooling.

This section characterizes equilibria. For notational simplicity, we drop the index for the first-period incentive scheme: s(c). In Proposition 1, we consider s(c) as given.

Proposition 1. For any first-period incentive scheme s(c), there exists no separating continuation equilibrium.

Proposition 1 shows that even when, in the static case, full separation is feasible and desirable (as in the case under assumption M), it is not even feasible in the dynamic case. The intuition behind the proof of this proposition is the following. If the agent fully reveals his information in the first period, he enjoys no second period rent (see Fact 1). Thus, he must maximize his first-period payoff. Now, suppose an agent with type $\beta$ deviates from his equilibrium strategy and produces at the same cost as if he had type $(\beta + d\beta)$, where $d\beta > 0$. From the envelope theorem, he loses only a second-order profit in the first period. On the other hand, he enjoys a first-order rent in the second period, because the principal is convinced he has type $(\beta + d\beta)$. Thus, he would like to pool with agent $(\beta+d\beta)$. The proof makes this intuition rigorous:

Proof of Proposition 1. Consider two types of firms: $\beta < \beta'$. Type $\beta$ produces at cost c and receives s. Type $\beta'$ produces at cost $c' \neq c$ and receives

s'. If the equilibrium is separating, c signals that the firm has type $\beta$; and similarly for $(\beta',c')$. So, in the second period, the firms are put at their individual rationality level, i.e., make a zero profit. Imagine that type $\beta'$ deviates and chooses to produce at cost c. In the second period this firm makes a zero profit, as the second-period incentive scheme is designed to extract all the surplus from the more efficient firm $\beta$. However, if firm $\beta$ deviates in the first period and produces at cost c', then it will make a strictly positive profit in the second period (since the less efficient firm makes a zero profit). We denote this profit $\Pi(\beta|\ \beta') > 0$. Optimization by both types of firms requires that:

$$(4.1) \qquad s - \phi(\beta-c) \geq s' - \phi(\beta-c') + \delta\Pi(\beta|\ \beta')$$

and

$$(4.2) \qquad s' - \phi(\beta'-c') \geq s - \phi(\beta'-c) \ .$$

Adding (4.1) and (4.2) we obtain:

$$(4.3) \qquad (\phi(\beta-c') + \phi(\beta'-c)) - (\phi(\beta-c) + \phi(\beta'-c')) > 0 \ .$$

Convexity of $\phi$ and (4.3) then imply that $c < c'$.

Thus, if $\{c(\beta),s(\beta)\}$ denotes the first-period allocation, c must be an increasing function of $\beta$. Therefore, c is differentiable almost everywhere. On the other hand, s must be decreasing (otherwise an agent of type $\beta$ would imitate an agent of type $\beta'$; $\beta' > \beta$), and therefore is differentiable almost everywhere. Consider now a point of differentiability $\beta$, say. If firm $\beta$ deviates and behaves like firm $(\beta-d\beta)$ $(d\beta > 0)$, it does not get a profit in the second period. Thus:

$$(4.4) \qquad s(\beta) - \phi(\beta-c(\beta)) \geq s(\beta-d\beta) - \phi(\beta-c(\beta-d\beta)) \ .$$

Taking the limit as $d\beta$ goes to zero:

$$(4.5) \qquad s'(\beta) + \phi'(\beta-c(\beta))c'(\beta) \geq 0 \ .$$

If firm $\beta$ deviates and behaves like firm $(\beta+d\beta)$, it obtains a second period surplus. Although its magnitude turns out to be irrelevant, this second period surplus is easy to compute: in the second period, firm $\beta$ must mimic firm $(\beta+d\beta)$'s outcome. Thus it saves $d\beta$ on effort. As the planner (believes he) has complete information on the firm, the marginal disutility of effort is equal to one. Thus $\Pi(\beta \mid \beta+d\beta) = d\beta$. We thus obtain

$$(4.6) \qquad s(\beta) - \psi(\beta - c(\beta)) \geq s(\beta+d\beta) - \psi(\beta - c(\beta+d\beta)) + \delta d\beta$$

or by taking the limit as $d\beta$ goes to zero:

$$(4.7) \qquad 0 \geq s'(\beta) + \psi'(\beta - c(\beta))c'(\beta) + \delta \ ,$$

which contradicts (4.5).

Q.E.D.

Proposition 1 shows that at least <u>some</u> pooling is necessary: there exists no non-degenerate subinterval of $[\underline{\beta}, \bar{\beta}]$ over which separation occurs. The next proposition shows that, for small uncertainty ($(\bar{\beta}-\underline{\beta})$ small), the principal imposes "<u>much</u> pooling," in a sense defined below. At this stage, it is worth emphasizing that under assumption M, the principal induces full separation in the static case, even for small uncertainty.

We will say that a continuation equilibrium exhibits <u>infinite reswitching</u> if there exist two equilibrium cost levels $c^0$ and $c^1$ and an infinite ordered sequence in $[\underline{\beta}, \bar{\beta}]$ : $\{\beta_k\}_{k \in N}$ such that producing at cost $c^0$ (respectively, $c^1$) is an optimal strategy for $\beta_{2k}$ (respectively, $\beta_{2k+1}$) for all k. An equilibrium which exhibits infinite reswitching is, thus, very complex; in particular, there does not exist a well-ordered partition of the interval $[\underline{\beta}, \bar{\beta}]$ into subintervals such that every type in a given subinterval chooses the same cost level (partition equilibrium). An example of an equilibrium

exhibiting infinite reswitching will be provided in Appendix B.

We will say that, for a given (small) $\varepsilon$, a continuation equilibrium exhibits pooling over a large scale $(1-\varepsilon)$ if there exist a cost level c and two values $\beta_1 < \beta_2$ such that $(\beta_2-\beta_1)/(\bar{\beta}-\underline{\beta}) > 1-\varepsilon$, and c is an optimal strategy for types $\beta_1$ and $\beta_2$. In other words, one can find two types which are arbitrarily far apart and pool. Note that, of course, a full pooling equilibrium (all types choose the same cost target) involves pooling over a large scale (for $\varepsilon = 0$).

Let us now state Proposition 2. To this purpose, we consider a sequence of economies with fixed $\bar{\beta}$, and we let the lower bound of the interval $\underline{\beta}_n$ converge to $\bar{\beta}$ (the density is thus obtained by successive truncations of the initial density: $f_1^n(\beta) = f_1(\beta)/\left(1-F_1(\underline{\beta}_n)\right)$, defined on $[\underline{\beta}_n, \bar{\beta}]$).

Proposition 2 (small uncertainty): For any $\varepsilon > 0$, there exists $\beta_\varepsilon < \bar{\beta}$ such that for all n such that $\underline{\beta}_n > \beta_\varepsilon$, and for any first-period incentive scheme s(c), there exists no continuation equilibrium which yields the principal a higher payoff than his optimal full pooling contract and which:

either: a) involves less than a fraction $(1-\varepsilon)$ of firms producing at the same cost; or

b) does not exhibit both infinite reswitching and pooling over a large scale $(1-\varepsilon)$.

Thus, for small uncertainty, the equilibrium must either be almost a full pooling equilibrium (almost all types have the same cost target) or exhibit infinite reswitching (complexity) and pooling over a large scale.

Proof of Proposition 2: The starting point of the proof consists in noticing that when $\underline{\beta}_n$ tends to $\bar{\beta}$, the distortion in the principal's profit relative to full information and associated with the best (full) pooling scheme tends to 0.[7] Thus, to prove the proposition, it suffices to prove that the distortion remains finite as long as the continuation equilibrium does not satisfy either a) or b).

Consider a first-period incentive scheme s(c), and two distinct levels of cost $c^0$ and $c^1$ that belong to the equilibrium path (i.e., that are best strategies for some types of agent) for some n (we will delete the subscript n in what follows). Let $\beta^i (i = 0,1)$ denote the supremum of types $\beta$ for which playing $c^i$ is optimal, and who are still active in the second period (i.e., are willing not to exercise their exit option) when they play $c^i$. $\beta^i$ will be called the "supremum for $c^i$". $\beta^i$ does not obtain any second period rent when playing $c^i$ (see Fact 2). Assume for the moment that $\beta^0 > \beta^1$. Thus, firm $\beta^0$ does not obtain any second period rent when playing $c^1$ either. Letting $\Pi(\beta | c^i)$ denote firm $\beta$'s second-period rent when it has played $c^i$ in the first period, we have:

---

[7]For instance, the principal can require a cost target $c = \bar{\beta} - e^*$ and give transfer $s = \phi(e^*)$. The first-period distortion (which exceeds the second-period one) is equal to:

$$E\left[(1+\lambda)\left(\phi(e^*) - \phi\left\{e^* - (\bar{\beta}-\beta)\right\} + (\bar{\beta}-\beta)\right)\right.$$
$$\left. + \lambda\left(\phi(e^*) - \phi(e^* - (\bar{\beta}-\beta))\right)\right] \to 0, \text{ when } \underline{\beta}_n \text{ goes to } \bar{\beta}.$$

One can show that the optimal full pooling cost target c satisfies:

$$\int_{\underline{\beta}}^{\bar{\beta}} \phi'(\beta - c) f_1(\beta) d\beta = 1 - \frac{\lambda}{1+\lambda} \int_{\underline{\beta}}^{\bar{\beta}} (\phi'(\bar{\beta} - c) - \phi'(\beta - c)) f_1(\beta) d\beta$$

(4.8) $s(c^0) - \phi(\beta^0-c^0) \geq s(c^1) - \phi(\beta^0-c^1)$

and

(4.9) $s(c^1) - \phi(\beta^1-c^1) \geq s(c^0) - \phi(\beta^1-c^0) + \delta\Pi(\beta^1 \mid c^0)$ .

Adding (4.8) and (4.9), we get

(4.10) $\phi(\beta^0-c^1) - \phi(\beta^1-c^1) + \phi(\beta^1-c^0) - \phi(\beta^0-c^0) \geq \delta\Pi(\beta^1 \mid c^0)$ .

Firm $\beta^1$, if it chooses $c^0$, can always duplicate what firm $\beta^0$ does: thus, if $e(\beta^0 \mid c^0)$ denotes firm $\beta^0$'s effort in the second period, we have:

(4.11) $$\begin{aligned}\Pi(\beta^1 \mid c^0) &\geq \phi(e(\beta^0 \mid c^0)) - \phi(e(\beta^0 \mid c^0) - (\beta^0-\beta^1)) \\ &\geq \phi'(e(\beta^0 \mid c^0) - (\beta^0-\beta^1))(\beta^0-\beta^1) \\ &\geq k(\beta^0-\beta^1),\end{aligned}$$

using the convexity of $\phi$, and the assumption that $\phi'$ is bounded below by $k$. Combining (4.10) and (4.11), we obtain:

(4.12) $$\frac{\phi(\beta^0-c^1) - \phi(\beta^1-c^1) + \phi(\beta^1-c^0) - \phi(\beta^0-c^0)}{\beta^0-\beta^1} \geq k > 0 .$$

Let us now come back to the sequence of economies. Consider a sequence of costs and suprema of types that choose these costs: $(c_n^0, c_n^1, \beta_n^0, \beta_n^1)$. We want to show that in the limit, $c_n^0$ and $c_n^1$ must be "sufficiently far apart." Using a Taylor expansion as $\beta_n^0$ and $\beta_n^1$ are close to $\bar{\beta}$, and inequality (4.12), we get

(4.13) $\phi'(\bar{\beta}-c_n^1) - \phi'(\bar{\beta}-c_n^0) \geq k' > 0$ .

(4.13) implies that there can be at most one of the two cost levels that

belongs to the equilibrium path and that belongs to the interval $[\bar{\beta}-e^*-\zeta, \bar{\beta}-e^*+\zeta]$, where $(\bar{\beta}-e^*)$ is the optimal effort in the limit (when n goes to infinity) and $\zeta$ is a given strictly positive constant. Thus, the fraction of firms that choose the other cost level must be negligible if the distortion relative to the first best is to converge to zero (which must be the case if the equilibrium dominates the full pooling optimum).

More generally, we must allow for the possibility that the suprema of types playing some costs and still active in the second period be equal. (4-13) applies only when these suprema differ pair-wise. However, it tells us that, for a given n, there exists $\beta_n$ and a set $C_n$ of equilibrium cost levels such that the corresponding suprema for all these cost levels coincide and are equal to $\beta_n$, and these cost levels are chosen by a fraction $(1-\varepsilon)$ of types, where $\varepsilon$ can be taken arbitrarily small if the equilibrium dominates the full pooling optimum.

That the equilibrium must exhibit pooling on a large scale follows: since cost levels in $C_n$ are chosen by a fraction $(1-\varepsilon)$ of types and all cost levels have the same supremum, there exist at least one cost level which is an optimal strategy for two types sufficiently far apart.

Last, if the equilibrium does not exhibit infinite reswitching, for any two cost levels $c^0$ and $c^1$ in $C_n$, $c^0$ is strictly preferred to $c^1$ in an interval $(\beta, \beta_n)$ and $\beta_n$ cannot be the supremum for $c^1$. Hence, there can exist only one cost in $C_n$, which means that the equilibrium is, up to $\varepsilon$, a full pooling equilibrium.

Q.E.D.

Let us discuss Proposition 2. For small uncertainty, the principal must either impose, up to a fraction E of the firms, a cost target (full pooling) or resort to an equilibrium with an infinite amount of reswitching and still much pooling. Appendix B constructs such an equilibrium, which is depicted in Figure 1. The principal offers two {cost, transfer} pairs.

Insert Figure 1.

The two costs are c and $\tilde{c}$. Firms in $[\underline{\beta}, \tilde{\beta})$ strictly prefer c. Firms in $[\tilde{\beta}, \bar{\beta}]$ are indifferent between c and $\tilde{c}$ and randomize between these two costs. Appendix B shows that this first-period randomization can be chosen so that the principal's posterior, and thus, the firm's second-period rent, maintain the equality between U and $\tilde{U}$ over this interval. It can also be shown that c and $\tilde{c}$ can be chosen arbitrarily close (by choosing $\tilde{\beta}$ close to $\underline{\beta}$); hence, a priori this equilibrium need not be suboptimal for small uncertainty.

We have been unable to show that full pooling is optimal for small uncertainty. We would, however, like to argue that it has robustness properties that, in practice, are likely to make it preferred to its complex contenders. The latter needs a very fine knowledge of the game in order to create the right equilibrium configuration. A unique cost target, by contrast, will be more robust to small mistakes in the description of uncertainty. Note that Proposition 2 implies that if the regulator is constrained to induce a "simple" equilibrium (pooling or partition), he chooses to impose a cost target (pooling).

In the class of full pooling equilibria, the best cost target is easy to characterize (see footnote 7). If such a cost target is imposed, the firm's first-period effort decreases with efficiency (while it increases with efficiency in the static model). Furthermore, in spite of the ratchet ef

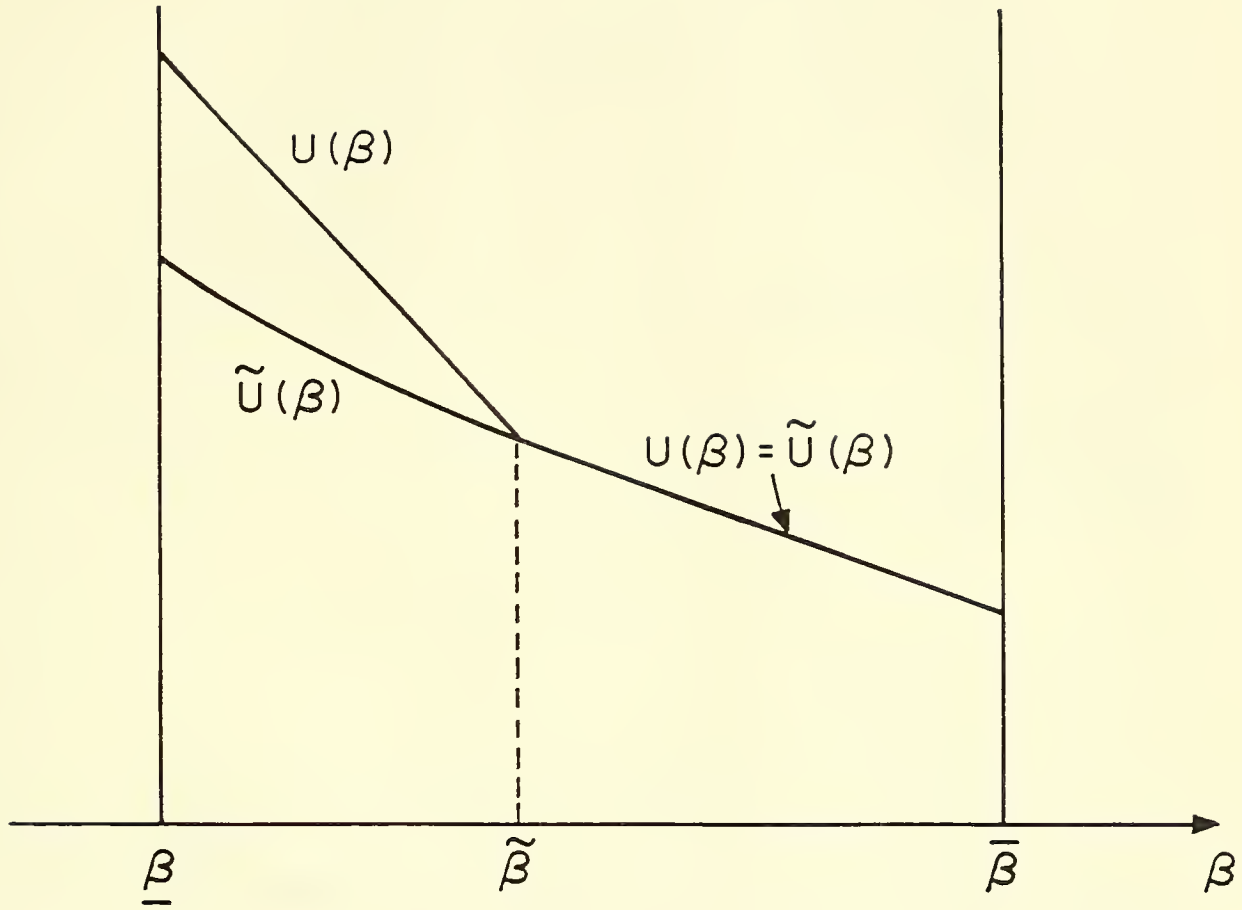


Figure 1 Non Partition Equilibrium

fect, there is no underprovision of effort in the first period. Indeed, it is possible to show that under a quadratic disutility of effort and a uniform prior, the average marginal disutility of effort over the population of types is the same as in the static (or full commitment) case. And, for $\beta$ close to $\bar{\beta}$, the firm works harder than in the first best! This is due to the fact that the regulator foresees this ratchet effect and forces the less efficient types to work very hard to avoid an excessive amount of shirking by the efficient types. Lastly, most efficient types work harder in the second period, while the least efficient types work harder in the first period. The variance of earnings ($s_t$) over the population of types grows over time (while it is constant under commitment). The optimal full pooling equilibrium in the uniform quadratic case is represented in Figure 2.

Insert Figure 2

5. Further results.

a) Partition vs. non-partition equilibria.

The natural type of equilibrium to look for in incentive problems is the partition equilibrium, in which $[\underline{\beta}, \bar{\beta}]$ can be divided into a (countable) number of ordered intervals such that in the first period all types in an interval choose the same cost level, and two types in two different intervals choose different cost levels (for examples of partition equilibria in sender-receiver games, see Crawford-Sobel [1982]). The case of full pooling is a degenerate partition equilibrium in which there is only one such interval.

We now derive necessary and sufficient conditions for the existence of a partition equilibrium when the disutility of effort is quadratic (or, more generally, $\alpha$-convex,[8] as the reader will notice).

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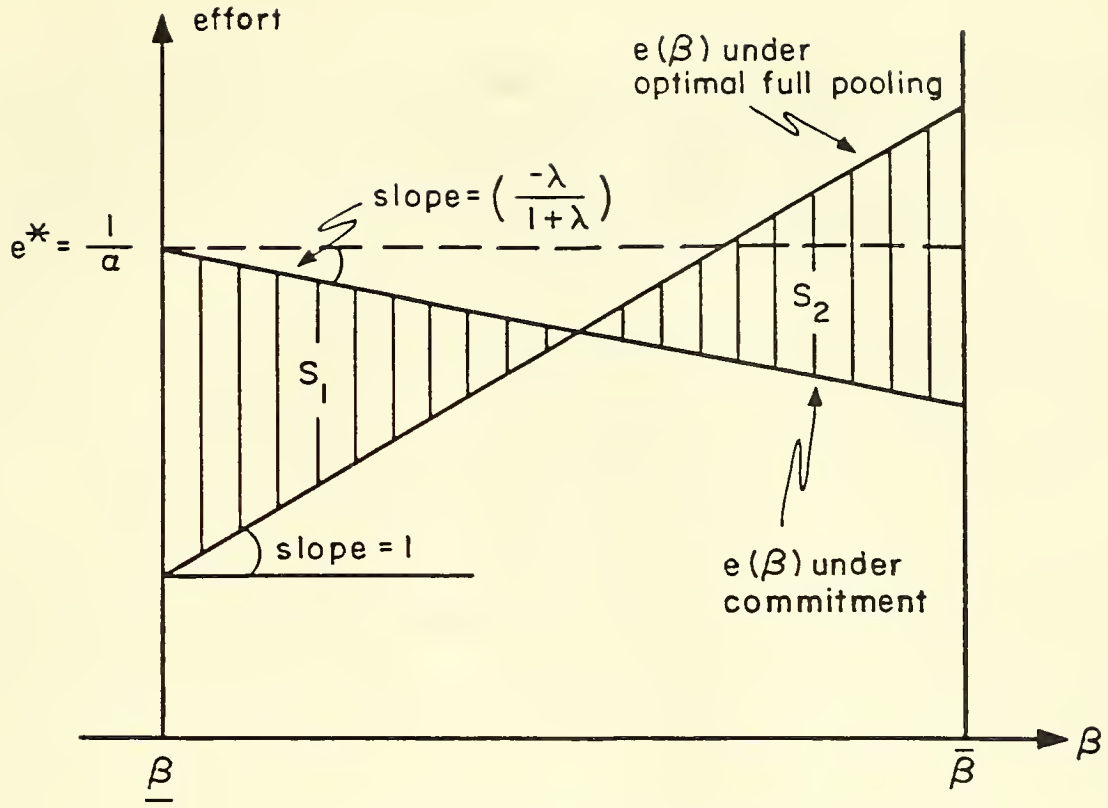
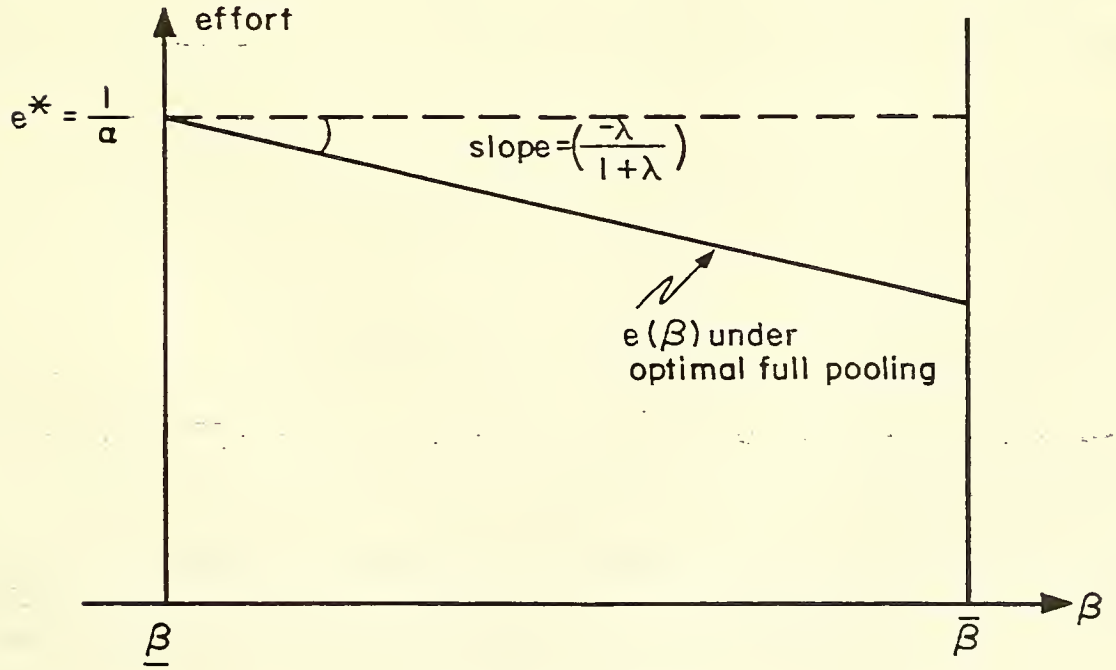
[8] $\psi$ is $\alpha$-convex if $\psi'' > \alpha$ everywhere.




First Period ( $S_1 = S_2$ )




Second Period

Figure 2. Quadratic-uniform case – small uncertainty

Proposition 3 (necessary condition). Assume $\psi$ is quadratic ($\psi(e) = \alpha e^2/2$), and the equilibrium is a partition equilibrium. If $c_k$ and $c_\ell$ are two equilibrium cost levels, $|c_k - c_\ell| \geq \delta/\alpha$.

Proposition 3 states that the minimum distance between two equilibrium costs in a partition equilibrium is equal to the discount factor divided by the curvature of the disutility of effort.

Proof of proposition 3. Let $\{\beta_k\}$ denote the cutoff points and $\{c_k\}$ the equilibrium costs in a partition equilibrium. Agents with types in $(\beta_k, \beta_{k+1})$ choose cost $c_k$ (where $\beta_k < \beta_{k+1}$). Agent $\beta_k$ is indifferent between $c_k$ and $c_{k-1}$. It is easily seen that $c_k$ increases with k: firm $\beta_{k+1}$ does not have a second-period rent when choosing $c_k$ or $c_{k-1}$ (from fact 2):

$$s(c_k) - \psi(\beta_{k+1} - c_k) \geq s(c_{k-1}) - \psi(\beta_{k+1} - c_{k-1}).$$

Also, firm $\beta_k$ does not enjoy a second-period rent when choosing $c_{k-1}$, but enjoys a strictly positive rent when choosing $c_k$. Thus,

$$s(c_{k-1}) - \psi(\beta_k - c_{k-1}) > s(c_k) - \psi(\beta_k - c_k).$$

Adding these two inequalities and using the convexity of $\psi$ leads to $c_k > c_{k-1}$.

Next, define the function $\Delta_k(\beta)$ on $[\beta_k, \beta_{k+1}]$:

$$\Delta_k(\beta) \equiv \{s(c_k) - \psi(\beta - c_k) + \delta\Pi(\beta | c_k)\} - \{s(c_{k-1}) - \psi(\beta - c_{k-1})\}.$$

To interpret $\Delta_k(\beta)$, remember that the principal's posterior beliefs about the agent's type when the agent chooses $c_{k-1}$ is the prior truncated on $[\beta_{k-1}, \beta_k]$. Thus, the agent with type $\beta_k$, and a fortiori with type $\beta > \beta_k$, enjoys no

second-period rent when he chooses $c_{k-1}$. When he chooses $c_k$, however, the posterior puts all the weight to $[\beta_k, \beta_{k+1}]$ and the agent enjoys a rent that we denote $\Pi(\beta|\, c_k)$. Thus, $\Delta_k(\beta)$ is the difference in intertemporal profits for agent $\beta$ in $[\beta_k, \beta_{k+1}]$ when he chooses $c_k$ and $c_{k-1}$. By definition, $\Delta_k(\beta_k) = 0$. If we want the agent with type $(\beta_k+\varepsilon)$ to choose $c_k$, it must be the case that $\Delta_k'(\beta_k) \geq 0$, or;

$$(5.1) \qquad -\psi'(\beta_k-c_k) + \delta\Pi'(\beta_k|c_k) + \psi'(\beta_k-c_{k-1}) \geq 0 .$$

From Fact 3 ("no distortion at the top" result), we know that $\Pi'(\beta_k|\, c_k) = -1$. Thus, a necessary condition is:

$$(5.2) \qquad \alpha(c_k-c_{k-1}) - \delta \geq 0.$$

Q.E.D.

Proposition 4 (sufficient condition). Assume $\psi$ is quadratic $(\psi(e) = \alpha e^2/2)$ and assumption M is satisfied. If the principal offers a finite set of allowed cost-transfer pairs $\{s_k, c_k\}$ such that $|c_k-c_\ell| \geq \delta/\alpha$ for all $(k,\ell)$, there exists a partition equilibrium.

The proof of Proposition 4 is provided in Appendix C. It is a constructive proof, which works by "backward induction" from $\bar{\beta}$. It starts by noticing that type $\bar{\beta}$, who never enjoys a second-period rent, maximizes its first-period profits. It then constructs the cut-off points of the partition equilibrium by moving towards $\underline{\beta}$.

We thus see that, contrary to the case in which sending a message is costless (as in Crawford-Sobel [1982]),[9] the existence of a partition equi-

[9]Here, the sender is the agent and the receiver the principal. The message is the first-period cost.

librium requires some stringent assumption. Indeed, it is possible to construct first-period incentive schemes for which there exists neither a partition equilibrium, nor a full-pooling (degenerate partition) equilibrium: see Appendix D.

b) The finite case

We have verified that the intuition and the characterization results obtained for the continuous case also hold for the large, but finite case (finite number of types for the agent and finite number of potential cost-transfer pairs for the principal). For instance, Proposition 1 can be stated informally:

Proposition 1': Let $[\underline{c},\bar{c}]$ denote an arbitrary cost range and $[\underline{\beta},\bar{\beta}]$ the uncertainty range. Assume that the set of potential efficiencies in the uncertainty range is finite, and that the principal is bound to offer a finite number of {cost, transfer} pairs, where the costs belong to the cost range. For a sufficiently large number of potential efficiencies, there exists no separating equilibrium.

Proof of Proposition 1': see Appendix E.

Thus, with a finite number of types, separation may be feasible, but as the grid becomes fine, equilibrium costs must go to $-\infty$ or $+\infty$ to allow separation. This clearly cannot be optimal for the principal. We checked that

(the natural analogs of) Propositions 2, 3, and 4 also hold in the finite case, but their statements are cumbersome.

For comparison, we should mention that we solved the two-type case. Even under our assumptions, incentive constraints are, in general, binding "upwards" and "downwards" (contrary to the static case). This leads to several potential equilibrium configurations. For instance there can exist continuation equilibria that exhibit "double randomization": Both types randomize between two cost levels in the first period (in the spirit of the continuation equilibrium exhibited in Appendix B). We were able to obtain the optimal scheme numerically. But more generally, the number of potential equilibrium configurations grows rapidly with the number of types, and a numerical analysis becomes hard to perform.

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## Appendix A

## Characterization of the Optimal Static Incentive Scheme

Let us assume that the principal has beliefs $dF(\beta)$ on the interval $[\underline{\beta},\bar{\beta}]$ ($dF$ need not be the truncated prior).

Let us first inquire about what type of function $e^*(\beta)$ can be implemented. The transfer $s$ is a function of $c = \beta - e^*(\beta)$ only. If we want to implement $e^*(\beta)$ we must have

$$(A.1) \qquad s(\beta-e^*(\beta))-\psi(e^*(\beta)) \geq s(\beta-e)-\psi(e) \qquad \text{for all } e.$$

Consider $\beta$, $\beta'$ with $\beta > \beta'$. In particular, it must be the case that agent $\beta$ is better off by choosing $e^*(\beta)$ rather than the level of effort which would identify him with agent $\beta'$, i.e.,

$$(A.2) \qquad e^*(\beta'|\beta) = \beta-\beta'+e^*(\beta')$$

(A.1) becomes in this case

$$(A.3) \qquad s(\beta-e^*(\beta))-\psi(e^*(\beta)) \geq s(\beta'-e^*(\beta'))-\psi(\beta-\beta'+e^*(\beta')) \ .$$

Reasoning symmetrically we obtain

$$(A.4) \qquad s(\beta'-e^*(\beta'))-\psi(e^*(\beta')) \geq s(\beta-e^*(\beta))-\psi(\beta'-\beta+e^*(\beta)) \ .$$

Adding (A.3) and (A.4) we have:

$$(A.5) \qquad \psi(e^*(\beta'))-\psi(\beta-\beta'+e^*(\beta')) \leq -\psi(e^*(\beta))+\psi(\beta'-\beta+e^*(\beta)) \ .$$

The convexity of $\psi$ then implies that $c(\beta)$ is nondecreasing and, therefore, differentiable almost everywhere.

The transfer $s$ must be a nonincreasing function of $c$ because it is always possible for an agent $\beta$ with a low cost (in terms of disutility of effort) to imitate agents with a higher cost.

Consider a point of differentiability of the level of utility of agent $\beta$. Then:

$$(A.6) \qquad \Pi(\beta) = s(\beta-e^*(\beta))-\psi(e^*(\beta)) \ .$$

Since agent $\beta$ chooses $e$ by maximizing $\{s(c)-\psi(e)\}$ with respect to $e$, we have

$$(A.7) \qquad -\frac{ds}{dc} - \psi'(e) = 0$$

$$(A.8) \qquad \dot{\Pi}(\beta) = \frac{ds}{dc}\left(1-\dot{e}^*(\beta)\right)-\psi'\left(e^*(\beta)\right)\dot{e}^*(\beta) = -\psi'\left(e^*(\beta)\right)$$

from (A.7).

The individual rationality constraint is

$$(A.9) \qquad \Pi(\beta) \geq 0 \qquad \text{for all } \beta.$$

The second period optimization problem of the regulator can be written:

$$(A.10) \qquad \text{Max} \int_{\underline{\beta}}^{\bar{\beta}} \{u-(1+\lambda)(\beta-e+\psi(e))-\lambda\Pi\}dF(\beta)$$

$$\text{s.t.} \qquad \dot{\Pi}(\beta) = -\psi'(e) \qquad dF. \text{ ae}$$

$$\dot{e}(\beta) = \xi \qquad dF. \text{ ae}$$

$$\Pi(\beta) \geq 0 \qquad \text{for all } \beta$$

$$\xi \leq 1 .$$

In this optimal control problem $\xi$ is the control and $\Pi$ and $e$ the state variables. Without restrictions on the shape of $F(\cdot)$, the constraint $\xi \leq 1$ may be binding in some intervals (i.e., the solution involves bunching) and the solution to (A.10) requires the use of methods described in Guesnerie and Laffont (1984).

Let us assume for the moment that $F(\cdot)$ is twice continuously differentiable and is such that $\xi \leq 1$ is never binding.

Then (A.10) can be rewritten more simply

$$(A.11) \qquad \text{Max} \int_{\underline{\beta}}^{\bar{\beta}} \{u-(1+\lambda)(\beta-e+\psi(e))-\lambda\Pi\}dF(\beta)$$

$$\text{st.} \quad \dot{\Pi}(\beta) = -\psi'(e) \qquad dF. \text{ ae}$$

$$\Pi(\beta) \geq 0 \qquad \text{for all } \beta,$$

where e is now the control variable and $\Pi$ the state variable.

The Hamiltonian for this problem is:

$$(A.12) \qquad H = \left[u-(1+\lambda)\left[\beta-e+\phi(e)\right)-\lambda\Pi\right] \frac{dF}{d\beta} - u\phi'(e)$$

(where $u(\beta) \geq 0$).

The first order conditions (these conditions are sufficient for (A.11) because the maximized Hamiltonian is concave in $\Pi$) can be written:

$$(A.13) \qquad (1+\lambda)(1 - \phi'(e)) \frac{dF}{d\beta} = u\phi''(e) \text{ ae}$$

$$(A.14) \qquad \dot{u} = \lambda \frac{dF}{d\beta} \quad \text{ae}$$

Since $\Pi$ is nonincreasing, $\Pi(\beta) \geq 0$ will be binding only at $\bar{\beta}$ and therefore, $u(\underline{\beta}) = 0$.

Thus, we have:

$$(A.15) \qquad \phi'(e) = 1 - \frac{\lambda}{1+\lambda} \frac{F(\beta)}{f(\beta)} \phi''(e) \ .$$

For example, if the distribution F is uniform and $\phi = \frac{e^2}{2}$,

$$u = \frac{\lambda(\beta-\underline{\beta})}{\bar{\beta}-\underline{\beta}}$$

$$e = 1 - \frac{\lambda}{1+\lambda} (\beta-\underline{\beta})$$

More generally, define

$$\underset{\sim}{\beta} \equiv \inf\{\beta | F(\beta+\varepsilon) > 0 \text{ for all } \varepsilon > 0\} \ .$$

$\underset{\sim}{\beta}$ is the real lower bound of the support of F. The following is the traditional "no distortion at the bottom" result (Fact 3):

Lemma 1. $\frac{d\Pi}{d\beta} (\underset{\sim}{\beta}) = -\phi'\left(e(\underset{\sim}{\beta})\right) = -1$ .

Proof. This results from the first-order condition and the fact that the ignored constraint clearly is not binding: (F/f) is close to zero and non-decreasing around $\underset{\sim}{\beta}$.

Q.E.D.

Next, note that if $\left(\frac{F}{f}\right)$ is non-decreasing and $\lambda$ is "not too big," (A.15) and the terminal condition $\phi'(e(\underset{\sim}{\beta})) = 1$ yield a solution $e(\beta)$ which decreases with $\beta$. This in particular implies that the ignored constraint ($\dot{e} \leq 1$) is indeed non binding. Thus, we obtain:

Lemma 2: If $\left(\frac{F}{f}\right)$ is non-decreasing, f is bounded away from zero and continuously differentiable, and $\lambda$ is "not too big," the marginal disutility of effort is non-increasing and does not exceed one (the first best level).

The following two properties hold for all distributions dF. Let $\tilde{\beta} \leq \bar{\beta}$ be defined by

$$\tilde{\beta} \equiv \sup\{\beta | F(\beta-\varepsilon) < 1 \text{ for all } \varepsilon > 0\} .$$

$\tilde{\beta}$ is the "real" upper bound of the interval.

Lemma 3. $\Pi(\tilde{\beta}) = 0$ .

Proof. We know that $\Pi(\tilde{\beta}) \geq 0$. Imagine that $\Pi(\tilde{\beta}) = \Pi > 0$. Reduce the transfer by $\Pi$ uniformly over all costs. Since $\Pi(\beta)$ is a decreasing function of $\beta$, all types that are at least as efficient as $\tilde{\beta}$ are willing to participate. The other types (which have measure zero) can always guarantee themselves zero by not participating. The uniform reduction in transfer is socially desirable.

Q.E.D.

Lemma 3 is straightforwardly extended to the case in which the cut-off point is lower than $\tilde{\beta}$ (some firms with types lower than $\tilde{\beta}$ do not produce). This results in Fact 2.

Lastly, let us show that, under assumption M, there exists an optimal incentive scheme such that an unexpected type $\beta < \underline{\beta}$ chooses to mimic agent $\underline{\beta}$. Assume that all cost levels that differ from the set $\{\tilde{\beta}-e^*(\tilde{\beta})\}_{\tilde{\beta} \in [\underline{\beta},\bar{\beta}]}$ are

prohibited (i.e., correspond to huge negative transfers). Then, agent $\beta$ can only duplicate an equilibrium cost. But, we know that:

(A.16) $$s(\underline{\beta}-e^*)-\psi(e^*) \geq s(\underline{\beta}-e)-\psi(e)$$

for all e such that there exists $\tilde{\beta}$ in $[\underline{\beta},\bar{\beta}]$ such that $\underline{\beta}-e = \tilde{\beta}-e^*(\tilde{\beta})$. If assumption M holds, $e^*(\tilde{\beta}) \leq e^*$, and therefore, $e \leq e^*$. The last step of the proof consists in noticing that (A.16), the convexity of $\psi$, and the fact that $e \leq e^*$ imply that:

(A.17) $$s(\beta-e^*)-\psi\left(e^*-(\underline{\beta}-\beta)\right)$$

$$> s\left(\tilde{\beta}-e^*(\tilde{\beta})\right)-\psi\left(e^*(\tilde{\beta})-(\tilde{\beta}-\beta)\right) .$$

Thus, the last part of Fact 4 is demonstrated.

Q.E.D.

* * *

## Appendix B

## Example of a Non-Partition Equilibrium

The non-partition equilibrium we construct is depicted in Figure 1.

The first-period incentive scheme offers two levels of cost $c < \tilde{c}$, and associated transfers: $s < \tilde{s}$. Let $\Pi(\beta|c)$ and $\Pi(\beta|\tilde{c})$ denote firm $\beta$'s second period rent when it has chosen c or $\tilde{c}$ in the first period. And let

$$U(\beta) \equiv s-\psi(\beta-c)+\delta\Pi(\beta|c) .$$

and

$$\tilde{U}(\beta) \equiv \tilde{s}-\psi(\beta-\tilde{c})+\delta\Pi(\beta|\tilde{c}) .$$

Assume that $\phi$ is quadratic: $\phi(e) = e^2/2$, that $\bar{\beta}-\underline{\beta} = 1$ and that the prior $f_1$ is uniform on $[\underline{\beta},\bar{\beta}]$: $f_1 = 1$.

The continuation equilibrium we construct has the following property: firms in $[\underline{\beta},\tilde{\beta}]$ produce at cost c. Firms in $[\tilde{\beta},\bar{\beta}]$ are indifferent between producing at c and producing at $\tilde{c}$. They randomize in such a way that the posterior distributions on $[\tilde{\beta},\bar{\beta}]$ given c and $\tilde{c}$ are uniform on this interval.

Let us introduce some more notation before constructing the equilibrium. Let g and $\tilde{g}$ denote the unconditional densities of firms choosing c and $\tilde{c}$: $g(\beta)+\tilde{g}(\beta) = 1$ for all $\beta$. And let f and $\tilde{f}$ denote the conditional (posterior) densities given that c and $\tilde{c}$ have been chosen in the first period. Since the densities are uniform, we have:

If $\beta \in [\underline{\beta},\tilde{\beta}]$ $\qquad f(\beta) = \dfrac{1}{(\tilde{\beta}-\underline{\beta})+(\bar{\beta}-\tilde{\beta})g}$

$$\tilde{f}(\beta) = 0 \ .$$

where g is the (uniform) density on $[\tilde{\beta},\bar{\beta}]$ of firms choosing cost c.

If $\beta \in [\tilde{\beta},\bar{\beta}]$ $\qquad f(\beta) = \dfrac{g}{(\tilde{\beta}-\underline{\beta})+(\bar{\beta}-\tilde{\beta})g}$

$$f(\beta) = \frac{1}{\bar{\beta}-\tilde{\beta}} \ .$$

We now put conditions on the parameters g (the uniform unconditional density given c is chosen), $\tilde{\beta}$ and the costs and transfers levels, so that this is indeed an equilibrium. These conditions are:

(B.1) $\qquad s-\phi(\bar{\beta}-c) = \tilde{s}-\phi(\bar{\beta}-\tilde{c})$

(B.2) $\qquad \tilde{c}-c = \dfrac{\delta\lambda}{1+\lambda}\left(\dfrac{\tilde{\beta}-\underline{\beta}}{g}\right) \ .$

Condition (B.1) says that firm $\bar{\beta}$ is indifferent between the two cost levels (remember that firm $\bar{\beta}$ never has any second-period rent). Condition (B.2)

insures that the indifference between costs c and $\tilde{c}$ is kept from $\bar{\beta}$ to $\tilde{\beta}$, as we now show. Let $e(\beta)$ and $\tilde{e}(\beta)$ denote firm $\beta$'s second-period effort when it has chosen cost c or $\tilde{c}$.

Let us show that, for all $\beta$ in $[\tilde{\beta},\bar{\beta}]$, $U(\beta) = \tilde{U}(\beta)$. Given (B.1), it suffices to show that, for all $\beta$ in $[\tilde{\beta},\bar{\beta}]$, $\dot{U}(\beta) = \dot{\tilde{U}}(\beta)$, or

$$(B.3) \qquad -\phi'(\beta-c)-\delta\phi'(e(\beta)) = -\phi'(\beta-\tilde{c})-\delta\phi'(\tilde{e}(\beta)) ,$$

where we use the fact that the derivative of the second-period rent is equal to minus the marginal disutility of effort (see section 2). Next, the posterior densities satisfy condition M on $[\underline{\beta},\bar{\beta}]$. From Appendix A and using the fact that $\phi$ is quadratic, we know that:

$$\phi'(e(\beta)) = 1 - \frac{\lambda}{1+\lambda}\,\frac{F(\beta)}{f(\beta)}$$

and

$$\phi'(\tilde{e}(\beta)) = 1 - \frac{\lambda}{1+\lambda}\,\frac{\tilde{F}(\beta)}{\tilde{f}(\beta)} ,$$

where F and $\tilde{F}$ are the cumulative distributions corresponding to f and $\tilde{f}$; on $[\tilde{\beta},\bar{\beta}]$:

$$F(\beta) = \frac{(\tilde{\beta}-\underline{\beta})+(\beta-\tilde{\beta})g}{(\tilde{\beta}-\underline{\beta})+(\bar{\beta}-\tilde{\beta})g}$$

and

$$\tilde{F}(\beta) = \frac{\beta-\tilde{\beta}}{\bar{\beta}-\tilde{\beta}} .$$

So, on $[\tilde{\beta},\bar{\beta}]$:

$$\phi'(e(\beta)) = 1 - \frac{\lambda}{\lambda+1}\left(\left(\frac{\tilde{\beta}-\underline{\beta}}{g}\right)+(\beta-\tilde{\beta})\right)$$

and

$$\phi'(\tilde{e}(\beta)) = 1 - \frac{\lambda}{1+\lambda}\,(\beta-\tilde{\beta}) .$$

(B.3) then becomes (using the fact that $\phi$ is quadratic):

$$\delta \frac{\lambda}{1+\lambda} \frac{\tilde{\beta}-\underline{\beta}}{g} = \tilde{c}-c \ ,$$

which is nothing but condition (B.2).

To complete the proof that this is indeed an equilibrium, we must first show that the parameters can be chosen so that g is less than 1, and also that for $\beta$ in $[\underline{\beta},\tilde{\beta}]$, $U(\beta) > \tilde{U}(\beta)$.

It is easy to ensure that $g = \frac{\delta\lambda}{1+\lambda} \frac{\tilde{\beta}-\underline{\beta}}{\tilde{c}-c}$ is less than one. It suffices to take $\tilde{\beta}$ close to $\underline{\beta}$. To check that firms in $[\underline{\beta},\tilde{\beta}]$ prefer to choose c, it suffices to show that, on this interval, $\dot{U}(\beta) \leq \dot{\tilde{U}}(\beta)$, or

$$-\phi'(\beta-c)-\delta\phi'(e(\beta)) \leq -\phi'(\beta-\tilde{c})-\delta\phi'(\tilde{e}(\beta)) \ .$$

Since condition M is satisfied for the posterior distribution given c, we have, for $\beta$ in $[\underline{\beta},\tilde{\beta}]$:

$$\phi'(e(\beta)) = 1 - \frac{\lambda}{1+\lambda} (\beta-\underline{\beta}) \ .$$

On the other hand, $\tilde{e}(\beta) = \tilde{e}(\tilde{\beta})-(\tilde{\beta}-\beta)$ (in the second period, $\beta$, which is to the left of the real lower bound $\tilde{\beta}$ of the posterior distribution, does produce at the same cost as $\tilde{\beta}$). So:

$$\phi'(\tilde{e}(\beta)) = \phi'(\tilde{e}(\tilde{\beta})-(\tilde{\beta}-\beta)) \ .$$

Using the fact that $\phi$ is quadratic, the condition for $\dot{U}(\beta) \leq \dot{\tilde{U}}(\beta)$ becomes, for any $\beta$ in $[\underline{\beta},\tilde{\beta}]$:

$$\tilde{c}-c \geq \delta\left(\frac{\lambda}{1+\lambda}\right) (\tilde{\beta}-\underline{\beta}) \ ,$$

which is satisfied from condition (B.2) and the fact that $g < 1$.

Q.E.D.

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## Appendix C
## Proof of Proposition 4 (sufficient conditions for the existence of a partition equilibrium)

Let $c^0$ denote the $\arg\max_c \{s(c)-\phi(\bar{\beta}-c)\}$ where $c \in \Gamma$, the set of allowed costs. If there are ties, take $c^0$ to be the lowest of the arg max. Firm $\bar{\beta}$ chooses $c^0$, since whatever the planner's beliefs in period 2, it will get a zero surplus in that period. Let $\beta^0 = \bar{\beta}$.

Let $\Pi(\beta|[\beta,\beta^0])$ denote firm $\beta$'s second-period profit when the planner's posterior distribution is the prior distribution truncated at $\beta$ (i.e., the planner knows that the firm's type belongs to the interval $[\beta,\bar{\beta}]$). $\Pi$ is continuous in $\beta$, and is equal to 0 for $\beta = \beta^0$ (from Fact 2). Define the function $h^0(\beta)$:

$$(C.1) \qquad h^0(\beta) \equiv s^0-\phi(\beta-c^0)+\delta\Pi(\beta|[\beta,\beta^0]) - \max_{c<c^0} \{s(c)-\phi(\beta-c)\} .$$

We know that $h^0$ is continuous and that from the definition of $c^0$, $h^0(\beta)$ is strictly positive for $\beta$ close (or equal) to $\beta^0$. Let $\beta^1 \equiv \max \{\beta|h^0(\beta) = 0 \text{ and } h^0(\beta-\varepsilon) < 0 \text{ for any sufficiently small } \varepsilon > 0\}$, and let $c^1$ denote the corresponding cost (as before, in case of ties, choose the lowest such cost). If there exists no such $\beta^1$ above $\underline{\beta}$ or if there exists no $c < c^0$ so that $h^0$ is not defined, then the equilibrium is a pure pooling one. Assume that $\beta^1 > \underline{\beta}$.

Let us notice that if in (C.1) we maximized over $c \geq c^0$ rather than over $c < c^0$, $h^0(\beta)$ would always be positive: we have:

$$s^0-\phi(\beta-c^0)+\delta\Pi(\beta|[\beta,\beta^0]) \geq s^0-\phi(\beta-c^0) .$$

But, for all c, from the definition of $c^0$:

$$s^0-\phi(\beta^0-c^0) \geq s(c)-\phi(\beta^0-c) .$$

Using $\beta \leq \beta^0$ and $c \geq c^0$ and the convexity of $\phi$, then leads to

$$s^0-\phi(\beta-c^0) \geq s(c)-\phi(\beta-c) \ .$$

Thus, in our quest for $\beta^1$, we can restrict ourselves to costs under $c^0$. This property (with the same proof) will hold at each stage of our algorithm.

Next, define the function $h^1(\beta)$:

$$h^1(\beta) \equiv s^1-\phi(\beta-c^1)+\delta\Pi(\beta|[\beta,\beta^1]) - \max_{c<c^1} \{s(c)-\phi(\beta-c)\}$$

$h^1$ is continuous; and from the construction of $c^1$, $h^1$ is strictly positive for $\beta$ slightly under $\beta^1$. Let $\beta^2 \equiv \max \{\beta<\beta^1 | h^1(\beta) = 0$ and $h^1(\beta-\varepsilon) < 0$ for any sufficiently small $\varepsilon\}$, and let $c^2$ denote the corresponding cost (in case of ties, choose the lowest such cost).

$(\beta^k,c^k)$ is then constructed by induction until either there exists no $\beta^k \geq \underline{\beta}$ that satisfies $h^k(\beta^k) = 0$ and $h^k(\beta^k-\varepsilon) < 0$ for small $\varepsilon$, or there is not allowed cost level left.

The partition equilibrium we propose has type $\beta$ in $(\beta^{k+1},\beta^k)$ choose cost $c^k$ (the zero-probability cut-off types are indifferent between two cost levels). When cost c is played, that does not belong to the equilibrium path, Bayes rule does not pin posterior beliefs down. We will assume that the principal then believes that the agent has type $\underline{\beta}$ (the reader who worries about the plausibiltiy of this conjecture should remember that the principal is always free not to allow such cost levels, so that the problem does not arise).

Let us first show that a type $\beta$ in $[\beta^{k+1},\beta^k]$ does not prefer a cost $c < c^k$. From the construction of $\beta^{k+1}$, we know that

$$s^k-\phi(\beta^{k+1}-c^k)+\delta\Pi(\beta^{k+1}|[\beta^{k+1},\beta^k]) = \max_c \{s(c)-\phi(\beta^{k+1}-c)\} \ .$$

Now, define for $\beta \in [\beta^{k+1},\beta^k]$ ,

$$\Delta_k(\beta) \equiv s^k-\phi(\beta-c^k)+\delta\Pi(\beta|[\beta^{k+1},\beta^k]) - \max_{c\leq c^k} \{s(c)-\phi(\beta-c)\}$$

Thus, $\Delta_k(\beta^{k+1}) = 0$. Let us show that $\Delta_k'(\beta) > 0$:

$$\Delta_k'(\beta) = -\phi'(\beta-c^k)+\delta\Pi'(\beta|[\beta^{k+1},\beta^k])+\phi'(\beta-c) ,$$

for some $c < c^k$.

Next, $\Pi'(\beta|[\beta^{k+1},\beta^k])$ is equal to minus the marginal disutility of effort by agent $\beta$ in the second period. Since the posterior beliefs are the truncated prior on $[\beta^{k+1},\beta^k]$, assumption M (monotone hazard rate) is satisfied, as is easily checked. From Fact 4, the marginal disutility of effort is lower than 1. Thus:

$$\Delta_k'(\beta) > -\delta+\alpha(c^k-c) > 0 ,$$

since the distance between the allowed costs exceeds $\delta/\alpha$.

Second, we must show that a type $\beta$ in $[\beta^{k+1},\beta^k]$ does not prefer a cost $c > c^k$. We prove this by "backward induction" from $\bar{\beta}$. Let us suppose that we have shown that for any $\beta$ in $[\beta^k, \bar{\beta}]$, $\beta$ prefers to play its presumed optimal cost to playing a higher cost. (To start the induction, remember that this property holds on $[\beta^1,\bar{\beta}]$, by definition of $\beta^1$). Define on $[\beta^{k+1},\beta^k]$ and for $c > c^k$:

$$\Lambda_k(\beta,c) \equiv \{s^k-\phi(\beta-c^k)+\delta\Pi(\beta|[\beta^{k+1},\beta^k])\}$$

$$- \{s(c)-\phi(\beta-c)+\delta\Pi(\beta|c)\} ,$$

where $\Pi(\beta|c)$ is agent $\beta$'s second-period rent when it chooses cost $c$ in the first period. Notice that, by induction and from the fact that $c^k$ is an optimal strategy for $\beta^k$,

$$\Lambda_k(\beta^k,c) > 0 .$$

To prove our property, it suffices to show that $\Lambda_k'(\beta,c) < 0$ on $[\beta^{k+1},\beta^k]$. But, we have:

$$\Lambda_k'(\beta,c) < -\phi'(\beta-c^k)+\phi'(\beta-c)-\delta\Pi'(\beta|c) .$$

If c does not belong to the equilibrium path, from our updating rule, we have

$\Pi'(\beta|c) = 0$. If c belongs to the equilibrium path, $c = c^{\ell}$ with $\ell < k$, then

$$\Pi'(\beta|c) = -\phi'(e^*-(\beta^{\ell+1}-\beta)) \geq -1$$

(see Fact 4: the $\beta$-agent mimics the cost chosen by the lower possible type given the second-period posterior beliefs). In both cases, $\Pi'(\beta|c) \geq -1$, so that

$$A'_k(\beta,c) \leq \delta-\alpha(c-c^k) \leq 0 .$$

Thus, the agent with type $\beta$ does not want to choose $c > c^k$ either.

Q.E.D.

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## Appendix D

## Example of Inexistence of a Partition or Pooling Equilibrium

Let us assume that $\phi$ is quadratic: $\phi = e^2/2$, say, and that $\beta$ is uniformly distributed on $[1,2]$. Suppose that the principal offers two cost-transfer pairs $\{c_0,s_0]$ and $\{c_1,s_1\}$. Assume w.l.o.g. that $c_0 > c_1$. If $c_0-c_1 < \delta$, we know from Proposition 3 that there exists no (non degenerate) partition equilibrium. Let us now show that one can choose the parameters so that there exists no pooling equilibrium either. Assume that the transfers satisfy:

$$(D.1) \qquad s_0-\phi(2-c_0) = s_1-\phi(2-c_1)+\varepsilon ,$$

where $\varepsilon$ is strictly positive and small. Equation (D.1) says that type $\bar{\beta} = 2$ slightly prefers $c_0$ (remember that this type never enjoys a second-period rent). Thus, a full pooling equilibrium must be at cost $c_0$.

To give the full pooling equilibrium at $c_0$ its best chance, let us assume that when the agent plays $c_1$ (a zero probability event), the principal

believes he has type $\underline{\beta} = 1$, so that the agent does not enjoy a second-period rent. Let $\Pi(\beta|[1,2])$ denote agent $\beta$'s rent when the posterior coincides with the prior, and define:

$$\Delta(\beta) \equiv \{s_0-\psi(\beta-c_0)+\delta\Pi(\beta|[1,2])\}-\{s_1-\psi(\beta-c_1)\}$$

to be the difference in intertemporal payoffs for agent $\beta$. We know that $\Delta(2) = \varepsilon$. We have:

$$\Delta'(\beta) = (c_0-c_1)+\delta\Pi'(\beta|[1,2])$$

$$= c_0-c_1-\delta\left(1 - \frac{\lambda}{1+\lambda}(\beta-1)\right) ,$$

using the computation in Appendix A. In particular,

$$\Delta'(2) = c_0-c_1 - \frac{\delta}{1+\lambda} .$$

Thus, for $\delta > (c_0-c_1) > \delta/(1+\lambda)$, there exists $\varepsilon$ sufficiently small such that $\Delta(\beta)$ becomes negative to the left of 2. Then there is no pooling equilibrium at $c_0$ either.

Q.E.D.

* * *

## Appendix E

## Proof of Proposition 1'

Consider a finite number of costs $\{c_k\}$ which are chosen on the equilibrium path. And let

$\bar{\beta}_k$ = sup $\{\beta|\beta$ produces at cost $c_k$ and is active in the second period$\}$.

Note that $\bar{\beta}_k$ obtains a zero surplus in the second period if it plays $c_k$ (see Fact 2). So for all $(k,\ell)$, with obvious notation, we have:

(E.1) $$s_k-\psi(\bar{\beta}_k-c_k) \geq s_\ell-\psi(\hat{\beta}_k-c_\ell)$$

and

$$(E.2) \qquad s_\ell - \phi(\bar{\beta}_\ell - c_\ell) \geq s_k - \phi(\bar{\beta}_\ell - c_k) \ .$$

Adding (E.1) and (E.2) gives:

$$(E.3) \qquad \phi(\bar{\beta}_k - c_\ell) + \phi(\bar{\beta}_\ell - c_k) - \phi(\bar{\beta}_k - c_k) - \phi(\bar{\beta}_\ell - c_\ell) \geq 0$$

(E.3) and the convexity of $\phi$ imply that:

$$(E.4) \qquad \bar{\beta}_k < \bar{\beta}_\ell \rightarrow c_k \leq c_\ell \ .$$

So there is an increasing relationship between the cost levels chosen on the equilibrium path, and the supremum of the types that choose these costs.

Now, consider two "adjacent" levels of cost belonging to the equilibrium path: $c_k < c_{k+1}$. We have

$$(E.5) \qquad s_{k+1} - \phi(\bar{\beta}_{k+1} - c_{k+1}) \geq s_k - \phi(\bar{\beta}_{k+1} - c_k) \ .$$

Furthermore, we can refine (E.2): type $\bar{\beta}_k$, after deviating to $c_{k+1}$, can always mimic what type $\bar{\beta}_{k+1}$ does in the second period. Given that the latter has a zero surplus and that it makes some effort $e_2(\bar{\beta}_{k+1})$, we get:

$$(E.6) \qquad s_k - \phi(\bar{\beta}_k - c_k) \geq s_{k+1} - \phi(\bar{\beta}_k - c_{k+1}) + \delta[\phi(e_2(\bar{\beta}_{k+1})) - \phi(e_2(\bar{\beta}_{k+1}) - \Delta\beta_k)] \ ,$$

where $\Delta\beta_k \equiv \bar{\beta}_{k+1} - \bar{\beta}_k$ .

Adding (E.5) and (E.6) we have:

$$(E.7) \qquad -\phi(\bar{\beta}_k - c_k) + \phi(\bar{\beta}_{k+1} - c_k) - \phi(\bar{\beta}_{k+1} - c_{k+1}) + \phi(\bar{\beta}_k - c_{k+1}) \geq \delta[\phi(e_2(\bar{\beta}_{k+1})) - \phi(e_2(\bar{\beta}_{k+1}) - \Delta\beta_k)] \ .$$

Now assume that the equilibrium is a separating equilibrium. Then, from Fact 1, $e_2(\bar{\beta}_{k+1}) = e^*$, i.e., $\phi'(e_2(\bar{\beta}_{k+1})) = 1$. It is clear from (E.7) that the right hand side is of the first order in $\Delta\beta_k$. Thus, if $(c_{k+1} - c_k)$ is not bounded away from zero, the left hand side cannot exceed the right hand

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side (take a Taylor expansion for $\Delta s_k$ small). But, as the grid size goes to zero, the range of cost must become infinite.

Q.E.D.

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